

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Numerical Methods

Subject Code : 4SC04MTE1

Branch: B.Sc.(Mathematics, Physics)

Semester : 4

Date : 16/05/2016

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions (14)**
- a) Write Simpson's one third rule. (01)
 - b) Write n^{th} approximation of Newton-Raphson method. (01)
 - c) Give value of $f'(x)$ by Newton's forward interpolation formula. (01)
 - d) What is the value of h ? (01)
 - e) Newton-Raphson method has second order convergence. Is following statement is true or false? (01)
 - f) Write Picard's formula for $\frac{dy}{dx} = f(x, y)$. (01)
 - g) How many ways we can find the solution of $\frac{dy}{dx} = f(x, y)$ with $f(x_0) = y_0$. Give any two names of method. (01)
 - h) What is the value of $f''(x)$ in general by Stirling interpolation formula? (01)
 - i) Find the values of $\int_0^1 e^x dx$ with $h = 1/2$ by Trapezoidal rule. (02)
 - j) Give value of a & b such that root of $f(x) = 0$ lies between a & b , where $f(x) = x^2 + x - 5$. (02)
 - k) Give general formula for Modified Euler Method. (02)

Attempt any four questions from Q-2 to Q-8.

- Q-2 Attempt all questions (14)**
- a) Find a root of $f(x) = 3x - 6 - \log_{10} x$ using Iteration Method up to four decimal places. (07)
 - b) Evaluate $\int_0^{10} e^x dx$ by Weddle's rule with $h = 1$. (07)
- Q-3 Attempt all questions (14)**
- a) Prove that Newton-Raphson Method has second order convergence. (07)
 - b) Using Picard's Method, obtain solution up to the fifth approximation to the equation $\frac{dy}{dx} = x + y; y(0) = 1$. Also find $y(0.1)$. (07)



- Q-4** **Attempt all questions** (14)
- a) Given $y' = x^2 + y^2$, $y(0) = 1$. Determine $y(0.1)$ and $y(0.2)$ by Modified Euler Method. (07)
- b) Find a positive root of $f(x) = x - \cos x$ by False Position Method correct to three decimal places. (07)

- Q-5** **Attempt all questions** (14)
- a) Derive $f'(x)$ by Newton's Forward Interpolation Formula. (07)
- b) Compute $y(2)$ if $y(x)$ satisfies the equation $\frac{dy}{dx} = \frac{1}{2}(x + y)$ given $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$ and $y(1.5) = 4.968$. Using Milne's Method. (07)

- Q-6** **Attempt all questions** (14)
- a) Find $y'(x)$ and $y''(x)$ at $x = 6$. For the function $y = f(x)$ given in the table: (07)
- | | | | | | | |
|-----|--------|--------|--------|--------|--------|--------|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 2.7183 | 3.3210 | 4.0552 | 4.9530 | 6.0196 | 7.3891 |
- b) Obtain approximate value of the root of $x = \sin x + \frac{\pi}{2}$ by Graphical Method. (07)

- Q-7** **Attempt all questions** (14)
- a) Compute $f'(0.8)$ and $f''(0.8)$ using the following table: (07)
- | | | | | | |
|--------|---------|---------|---------|---------|---------|
| x | 0.4 | 0.6 | 0.7 | 0.9 | 1.2 |
| $f(x)$ | 1.08107 | 1.18546 | 1.25517 | 1.43309 | 1.81066 |
- b) Derive Newton-Raphson Iteration formula for $\sqrt[n]{R}$. Also find the value of $\sqrt[3]{5}$ using this formula. (07)

- Q-8** **Attempt all questions** (14)
- a) Using Runge-Kutta Method of fourth order, solve the following for $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ with $h = 0.1$. (07)
- b) The velocity v of a particle at distance's from point on its path is given by the following table: (07)

s (m)	0	10	20	30	40	50	60
v (m/s)	47	58	64	65	61	52	38

Find the time taken to travel 60 meter. Using Simpson's one-third rule.

